STEP MATHEMATICS 2

2018

Examiner's Report

General Comments

The pure questions were again the most popular of the paper, with only two of those questions being attempted by fewer than half of the candidates (none of the other questions was attempted by more than half of the candidates). Good responses were seen to all of the questions, but in many cases, explanations lacked sufficient detail to be awarded full marks. Candidates should ensure that they are demonstrating that the results that they are attempting to apply are valid in the cases being considered. In several of the questions, later parts involve finding solutions to situations that are similar to earlier parts of the question. In general candidates struggled to recognise these similarities and therefore spent a lot of time repeating work that had already been done, rather than simply observing what the result must be.

Question 1

This was the most popular question of the paper, attempted by 93% of candidates. While many candidates scored well on this question there were very few who achieved full marks and a significant number who scored 0 for this question.

The first two parts of the questions were generally done well, with the majority of marks being lost for arithmetic errors or from just considering one of the two cases. In some cases, candidates produced the reasoning for part (ii) as their answer for part (i). In these cases, the candidates generally repeated the work as their answer for part (ii). There were a number of solutions to part (ii) that were overcomplicated, involving consideration of the factorised form and comparison of coefficients.

Part (iii) was also very well completed by those who attempted it, with the main cause for loss of marks being algebraic errors. A number of errors were seen in the final part of the question, particularly not considering the second case or not realising that the quadratic would have a different discriminant in this case and a failure to check that the roots of the quadratic did not duplicate the repeated root already found in this case.

Question 2

This was the least popular of the pure questions on the paper, attempted by only 35% of the candidates. Very few candidates were able to score full marks and a fairly high proportion scored 0 on this question.

In the first part of the question, a significant number of candidates sketched convex functions and so had sketches that did not match the inequality. Most candidates were able to identify many of the key points required for the final section of this introductory part, but many could not explain it clearly enough for full marks in this section.

Most candidates made good attempts at the next section, but again the presentations often lacked enough clarity about how the inequalities were being linked together to be awarded full marks. In some cases, the choices for x_1 and x_2 were not within the interval (x_1, x_2) and so these candidates lost marks.

Candidates who attempted to complete the final parts of the question were generally confident about how the results from the first part were to be applied, but a significant number omitted to demonstrate that the function being used was concave and so were not able to achieve full marks here.

Question 3

The first part of this question was well attempted in general, although some solutions required more care to be taken to check whether or not the extreme values are included within the range or not.

Many solutions to part (ii) did not generally include very clear explanations of the method and candidates often did not make it clear that they had demonstrated the result in both directions.

For the final part, candidates often did not find the “certain point” referred to in the question and instead tried to work with a general point (a, b). Some candidates also attempted to calculate the integral directly rather than making use of the fact that the graph has rotational symmetry.

Question 4

This was the second most popular question on the paper. The average mark scored by candidates attempting this question was the highest of all the pure questions.

The first two parts of the question were generally well done with candidates showing confidence in applying the given identity and then factorising and solving the resulting equation. A sizeable minority struggled to deal with the given range for x and so were unable to find all the solutions. In part (ii) many candidates were able to explain clearly why $\cos x = \cos y$ leads to $x = y$.

Part (iii), however, proved to be difficult for the majority of candidates. A small number of candidates did manage to find the quadratic equation and most of these were able to proceed and complete the question fully. Most candidates did not score very many marks in this section.

Question 5

Of the pure questions this was the one that attracted the poorest responses in general, with a significant proportion of attempts scoring 0.

In the first part of the question the majority of candidates did not include the constant of integration and so did not produce a fully justified solution. The expansion and substitution in the second part of the question was done well in general, although many candidates attempted to expand the e^{-x} as well as the $(1 - e^{ax})$, after which they were unable to complete the integral. In many of these cases there were then unjustified jumps to the series expansion found in part (i).

Answers to part (iii) were generally much better than part (ii), although some substitution errors were seen. Most of the marks lost in this part were because candidates failed to spot the connection with the previous part. It is worth noting that many candidates did not attempt part (iii), having failed to complete part (ii) successfully. It is likely that some additional marks could have been scored by these candidates had they attempted this final part.

Question 6

Solutions on this question either scored very well or very poorly, depending on the quality of explanation provided by candidates in their solutions. In the weakest cases the only marks that were awarded were for finding some of the particular cases.

In the first part of the question candidates were generally able to find the cases that satisfied the equation, but many of the explanations that there are no solutions if $n \geq 5$ were not sufficiently well produced to receive full marks.

In the second part of the question many students just restated the theorems without explaining the reasoning that followed from them. The cases for small values of n were generally found, but some candidates struggled to find the pair $(4,10)$. Some candidates also did not attempt to explain why the cases $n = 5$ and $n = 6$ did not produce solutions.

There were some attempts to calculate the values of large factorials in this question. Candidates should be aware that such an approach will not be the correct method with which to tackle the questions.

Question 7

This was an unpopular question among the pure questions, with slightly less than two fifths of the candidates attempting it. Many of the responses did not progress far through the problem, although there were some excellent solutions seen. As a result, attempts at this question generally received either very few or most of the marks.

A range of different methods were used, such as consideration of ratios of areas, or parallel lines to the diagram and the use of similar triangles.

The most common difficulties arose from candidates not recognising that $\boldsymbol{m}$ was parallel to $\boldsymbol{a}$ and $\boldsymbol{n}$ was parallel to $\boldsymbol{b}$.

Question 8

This was the third most popular question on the paper and a number of very good responses were seen from candidates.

Most candidates were able to apply the given substitution to the differential equation and then separated the variables successfully. Candidates adopted a range of approaches to solving the differential equation, such as use of an integrating factor or a solution by finding a complementary function and then a particular integral. Success was seen with all of these methods. Some candidates omitted the constant of integration and then were unable to reach the correct answer.

In the second part of the question most candidates were able to spot a suitable substitution and proceeded to solve the differential equation successfully. The best candidates spotted the similarity to part (i) and therefore saved some time on this part by modifying the answer to (i) rather than working through all of the steps again.

The final part of the question was the least well answered. Although most candidates realised that $y_1(x) > y_2(x)$, many did not justify this or substituted a value to check rather than demonstrating that it was true for all values. Many candidates also failed to recognise that the gradient should be 0 at the origin for both curves.

Question 9

This was the most popular of the mechanics questions, with almost half of the candidates attempting it. Many candidates were able to apply the required sequence of calculations and secured good marks on this question, although in some cases some steps were omitted which limited the amount of success that could be achieved on the question. In the final part of the question weaker candidates struggled to get the signs of the velocities correct and therefore were unable to reach the correct coefficient of restitution between the two particles.

Question 10

The qualities of responses for this question were quite varied. Many candidates who struggled with this question failed to set up the speed of a point on the string in the first part of the question and were therefore unable to make any significant progress. In the part of the question considering the return journey a common error was to “restart the clock” once the ant had reached the endpoint, however, since the speed of the string is dependent on the time from the start of the problem this led to an error.

A particularly elegant solution to the final part involved changing the frame of reference such that the endpoint was stationary and the peg was moving at constant speed. Stronger candidates were then able to use the symmetry of the problem to reduce the amount of algebra needed considerably.

Question 11

The most common mistake made in this question was to have the frictional force acting in the wrong direction, with many candidates assuming that the frictional force was pulling the motorbike backwards and a “driving force” from the engine acted to push it forwards. The great majority of candidates did attempt to find moments about the centre of mass as instructed, but there were some attempts to evaluate moments about one of the wheels.

Question 12

This question was the most poorly attempted of all of the questions. While approximately one fifth of candidates attempted this question (more than questions 10 or 11), many of the solutions scored very low marks. The few candidates who were able to make progress on the question were however able to secure very good marks.

There were a number of candidates who clearly did not understand the payoffs described in this question, thinking for example that the process continued until a tail was reached and was then related to the number of heads achieved. Many of the students attempted differentiation to maximise the expected winnings, but often did not progress beyond the non-integer value found to check the two possible integer values.

In the second part many candidates struggled to find a useful method of counting successful outcomes, and therefore could not make much further progress on the question. In many cases the quality of explanation seen accompanying the method was not sufficiently detailed to demonstrate that a valid method was being attempted – candidates would be well advised to pay attention to the explanation of their method in questions such as this.

In the final part, many candidates failed to recognise the similarity between the function to be maximised and that from the first part of the question and therefore attempted to work through the process again. The manipulation of logarithms for the final part of the question was generally well done.

Question 13

This was the more popular of the two probability and statistics questions on the paper and many good responses were seen.

Candidates were generally able to work out the probabilities required in the first part of the question accurately. A considerable number of candidates were not able to make significant progress beyond that point, but those who did were often able to identify the relationships clearly and make use of the symmetry of the problem. Some attempts to tackle the second part of the question through counting arguments were seen, but these were not successful. The proportion of candidates achieving full marks for this question was higher than any other.